OPCIÓN A

1

$$A=\begin{pmatrix}0 & 2\\ 3 & 0\end{pmatrix} \qquad B=\begin{pmatrix}a & b\\ 6 & 1\end{pmatrix}$$

a) $A\cdot B = B\cdot A \Rightarrow \begin{pmatrix}0 & 2\\ 3 & 0\end{pmatrix}\cdot\begin{pmatrix}a & b\\ 6 & 1\end{pmatrix}=\begin{pmatrix}a & b\\ 6 & 1\end{pmatrix}\cdot\begin{pmatrix}0 & 2\\ 3 & 0\end{pmatrix} \Rightarrow \begin{pmatrix}12 & 2\\ 3a & 3b\end{pmatrix}=\begin{pmatrix}3b & 2a\\ 3 & 12\end{pmatrix}$

$\Rightarrow$ $3b=12 \Rightarrow \boxed{b=4}$ $\quad 2=2a \Rightarrow \boxed{a=1}$ $\quad 3b=12 \Rightarrow b=4$ $\quad 3a=3 \Rightarrow a=1$

b) $a=1$, $b=0$ $\qquad X\cdot B - A = I_2 \Rightarrow \begin{pmatrix}x & y\\ z & t\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 6 & 1\end{pmatrix}-\begin{pmatrix}0 & 2\\ 3 & 0\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\Rightarrow$

$\Rightarrow \begin{pmatrix}x+6y & y\\ z+6t & t\end{pmatrix}-\begin{pmatrix}0 & 2\\ 3 & 0\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\Rightarrow \begin{pmatrix}x+6y & y-2\\ z+6t-3 & t\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$\begin{cases} x+6y=1 \\ y-2=0 \rightarrow \boxed{y=2} \\ z+6t-3=0 \\ \boxed{t=1}\end{cases}$$

$x+12=1 \rightarrow \boxed{x=-11}$

$z+6-3=0 \Rightarrow \boxed{z=-3}$

$$X=\begin{pmatrix}-11 & 2\\ -3 & 1\end{pmatrix}$$

2

$$f(x)=\begin{cases}\dfrac{2x}{x-1} & \text{si } x<2\\ 2x^2-10x & \text{si } x\geq 2\end{cases}$$

a) $x-1=0 \Rightarrow x=1$ $(1<2) \Rightarrow$ $\text{Dom}(f)=\mathbb{R}-\{1\}$.

b) continuidad $x=2$:

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2}\frac{2x}{x-1}=\frac{4}{1}=4$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2} 2x^2-10x=8-20=-12$$

$\Rightarrow \nexists \lim_{x\to 2} f(x)$

f no es continua en $x=2 \Rightarrow f$ no es derivable en $x=2$.

c) Recta tangente a f en $x=0 \Rightarrow y-f(0)=f'(0)(x-0)$

$f(0)=\dfrac{2\cdot 0}{0-1}=0$ $\qquad f'(x)$ $(x<2) \rightarrow f'(x)=\dfrac{2\cdot(x-1)-2x\cdot 1}{(x-1)^2}=\dfrac{2x-2-2x}{(x-1)^2}=\dfrac{-2}{(x-1)^2}$

$f'(0)=\dfrac{-2}{(-1)^2}=-2$.

$y-0=-2(x-0) \Rightarrow \boxed{y=-2x}$ RECTA TG. A f EN $x=0$.

3 PARTE I

a) $P(A)=0'5$, $P(B)=0'4$, $P(A\cup B)=0'8$

$$P(A/B)=\frac{P(A\cap B)}{P(B)}$$

$P(A\cup B)=P(A)+P(B)-P(A\cap B) \Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cup B) \Rightarrow$

$\Rightarrow P(A\cap B)=0'5+0'4-0'8=0'1$.

$P(A/B)=\dfrac{0'1}{0'4}=0'25$

b) $P(C)=0'3$, $P(D)=0'8$

C y D independientes $\Rightarrow P(C\cap D)=P(C)\cdot P(D)=0'3\cdot 0'8=0'24$

$P(C\cup D)=P(C)+P(D)-P(C\cap D)=0'3+0'8-0'24=0'86$

PARTE II

X = nº días permanecía en un hospital

$X \sim N(\mu, \sigma) = N(\mu, 3)$

a) I.C. μ $\quad 1-\alpha = 0'97 \Rightarrow \alpha = 0'03 \Rightarrow \alpha/2 = 0'015 \Rightarrow z_{\alpha/2} = 2'17$ ($0'985$ ↑)

$n = 100$

$\bar{x} = 8'1$

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$$\left(8'1 - 2'17 \cdot \frac{3}{\sqrt{100}},\ 8'1 + 2'17 \cdot \frac{3}{\sqrt{100}}\right)$$

$$\left(8'1 - 2'17 \cdot 0'3,\ 8'1 + 2'17 \cdot 0'3\right)$$

$$\left(7'449,\ 8'751\right)$$

b) ¿n?

μ

Error máximo 1

$1-\alpha = 0'92 \Rightarrow \alpha = 0'08 \Rightarrow \alpha/2 = 0'04$

$z_{\alpha/2} = 1'75$ $\quad (0'96)$

Error ≤ 1

$$z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \leq 1$$

$$1'75 \cdot \frac{3}{\sqrt{n}} \leq 1$$

$$\sqrt{n} \geq \frac{1'75 \cdot 3}{1}$$

$$n \geq (1'75 \cdot 3)^2$$

$$n \geq 27'5625$$

Tema 28